RESISTANCE OF A PLANE PLATE IN A TURBULENT FLOW OF WEAK POLYMER SOLUTION

Yu. F. Ivanyuta and L. A. Chekalova UDC 532.517.4

The effectiveness of polymer solutions in flow within a tube and flow around a plate is compared. The calculated results obtained agree well with experimental data.

The problem of turbulent flow of a liquid over a plane plate is a quite simple one, but one which is very important in practice. In many cases of practical importance Reynolds numbers are so high that they cannot be realized in laboratory conditions, and even at moderate Reynolds numbers measurements in the boundary layer on the plate are more cumbersome than similar measurements undertaken for a flow within a tube. Thus, the method proposed by L. Prandtl and T. Von Karman, which permits calculation of the flow resistance of the plate from results obtained for flow in tubes, is of special interest. It is now possible to use this method to study the flow of weak polymer solutions along a plane plate because at the present time a large quantity of experimental data has been accumulated on the flow of such solutions in tubes.

It follows from such experimental studies [1, 2] that the velocity profile may have one of the limiting forms

$$\varphi = u/v_* = 5.75 \lg (yv_*/\nu) + 5.5, \quad (1)$$

$$\varphi = 26 \lg (yv_*/\nu) - 18.2 \quad (2)$$

or an intermediate form

$$\varphi = 5.75 \lg (yv_*/\nu) + (20.25 \lg \eta_0 - 18.2). \quad (3)$$

The parameter $\eta_0 = (y_1 v_*/\nu)$ is that value of the coordinate at which Eqs. (1) and (2) mesh.

From conditions (1), (2), (3) for maximum velocity values on the tube axis the dependence of fluid friction coefficient λ on Reynolds number Re can be obtained in the standard manner for a fixed value of the parameter η_0 ($\eta_0 > 15$, for water flow $\eta_0 \approx 15$). Data from such calculations (Fig. 1) permit determination of the decrease in drag reduction $S = (\lambda_W - \lambda_P)/\lambda_W$ as a function of the parameter $(\eta_0 - 15)$, numerically determining the difference in velocity profiles for water and polymer solution flow in a tube (Fig. 2).

It has been experimentally established that addition of a polymer to a water flow reduces its friction drag over some limited range of variation of shear stress values on the wall. The initial threshold stress depends only on the form of the polymer, and the other limit depends on concentration and surface roughness as well. Within this range there is a smooth increase in the effect to a maximum value corresponding to the chosen polymer concentration, after which the effect is maintained and then decreases smoothly [3]. For this portion of the range of the shear stress values on the wall, where the drag reduction remains constant for the chosen polymer concentration, there is good agreement between calculated data and experimental data of various authors on the dependence of the stress reduction on polymer concentration [S = f(c), Fig. 2]. In such a comparison each polymer is characterized by some constant k, g/m^3, which relates the concentration c to the parameter $(\eta_0 - 15)$ in a linear manner:

$$c = k(\eta_0 - 15). \quad (4)$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 36, No. 1, pp. 9-15, January, 1979. Original article submitted March 28, 1978.

 0022-0841/79/3601-0004$07.50

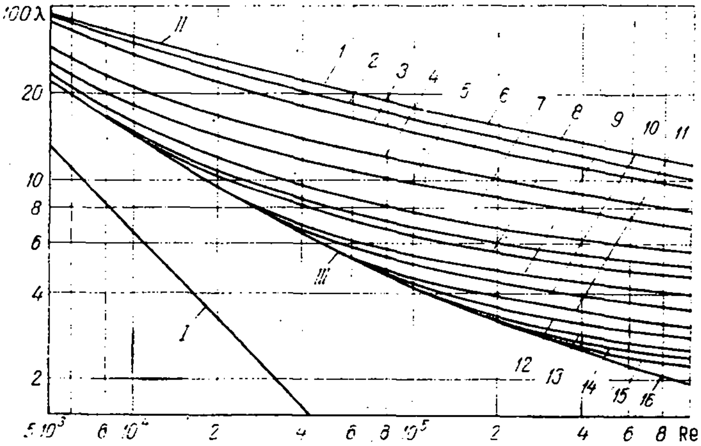

Fig. 1. Friction coefficient λ versus Reynolds number Re = vd/ν and parameter η_0 of velocity profile for flow of polymer solutions within a tube: I) λ = 64/Re (laminar flow of water); II) $1/\sqrt{\lambda} = 2 \log Re\sqrt{\lambda} - 0.8$ (turbulent flow of water); III) $1/\sqrt{\lambda} = 9.2 \log Re \sqrt{\lambda} - 19.6$ (limiting case of polymer solution flow): 1) η_0 = 15; 2) 17; 3) 20; 4) 30; 5) 40; 6) 60; 7) 80; 8) 100; 9) 150; 10) 200; 11) 300; 12) 400; 13) 600; 14) 800; 15) 1000; 16) 2000.

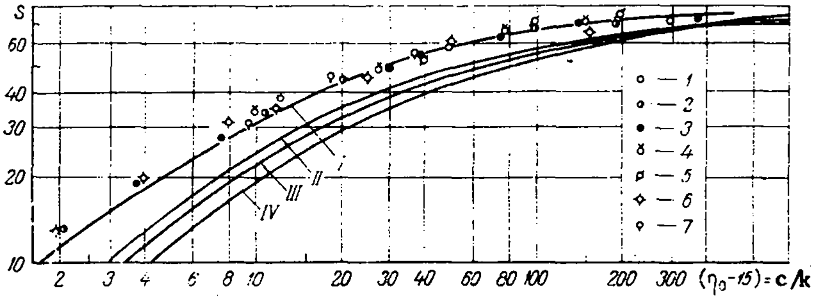

Fig. 2. Drag reduction S, %, versus form of velocity profile and concentration of polymer solution (experiment within tube) (η_0 − 15) = c/k: 1) polyox, k = 0.40 g/m^3 [1]; 2) same polymer, 0.05 g/m^3 [4]; 3) same, 0.13 g/m^3 [3]; 4) polyacrylamide, k = 0.50 g/m^3 [4]; 5) KMTs-7HSCP, k = 25 g/m^3 [7]; 6) guara resin, k = 12.6 g/m^3 [4]; 7) same polymer, 12.6 g/m^3 [6]. I) tube, Re = 10^5; II, III, IV) plate, Re = 10^7, 10^8, 10^9, respectively. Solid lines, calculation.

The coefficient k characterizes the effectiveness of the polymer and can serve as the parameter determining the ability of the given polymer to decrease friction drag in aqueous solution: for polyox WSR-301 k = (0.05–0.40) g/m^3 (depending on the term for which it is preserved), for polyacrylamide k = 0.5 g/m^3, for guara resin k = 12.6 g/m^3, and for KMTs-7HSCP k = 25 g/m^3.

On the basis of these principles of polymer flow in tubes considered above, the basic principles of flow of such solutions on a plate can be derived if the following assumptions are made, as was done by Prandtl: the boundary layer on the plate is turbulent, beginning at the forward edge; the maximum flow velocity on the tube axis corresponds to the velocity of the flow incident on the plate at infinity; and, the velocity distribution in the boundary layer on the plate is the same as in the tube.

Since the universal logarithmic law of velocity distribution for flow in a tube permits extrapolation to arbitrarily large Reynolds numbers, it can be expected that the resistance law for the plate which is to be derived will also admit extrapolation to arbitrarily large Reynolds numbers.

For velocity distributions in the most general form

$$\varphi = \varphi(\eta); \quad \varphi = u/v_*; \quad \varphi_N = U_\infty/v_*; \quad \eta = yv_*/\nu; \quad \eta_N = \delta v_*/\nu, \tag{5}$$

with use of the momentum theorem

$$\rho v_*^2 = \rho \frac{d}{dx}\left[\int_0^\delta u(U_\infty - u)\,dy\right] \tag{6}$$

the following relationships are valid [8]: Reynolds number

$$U_\infty L/\nu = \mathrm{Re} = \Phi(\eta_k) = \int_0^{\eta_k} \frac{d\varphi_N}{d\eta_k}\left(\int_0^{\eta_N} \varphi^2 d\eta\right) d\eta_N, \tag{7}$$

$$\eta_k = \delta v_*/\nu|_{x=L};$$

total friction coefficient

$$c_f = 2\Psi(\eta_k)/\Phi(\eta_k), \tag{8}$$

where

$$\Psi(\eta_k) = \int_0^{\eta_k} \frac{1}{\varphi_N^2} \frac{d\varphi_N}{d\eta_N}\left(\int_0^{\eta_N} \varphi^2 d\eta\right) d\eta; \tag{9}$$

local friction coefficient

$$c_f' = 2/\varphi_N^2. \tag{10}$$

For a universal logarithmic velocity distribution

$$\varphi = A \lg \eta + D \tag{11}$$

or

$$\varphi = a \ln(1 + b\eta), \tag{12}$$

where

$$A = 2.305a, \quad D = a \ln b, \tag{13}$$

and upon introduction of the new variable

$$z = 1 + b\eta \tag{14}$$

the functions $\Phi(\eta)$ and $\Psi(\eta)$ have the form [8]

$$\Phi(\eta) = \frac{a^2}{b}\,(z \ln^2 z - 4z \ln z - 2\ln z + 6z - 6),$$
$$\Psi(\eta) = \frac{a}{b}\left(z + 1 - \frac{2(z-1)}{\ln z}\right). \tag{15}$$

In flow of a polymer solution on a plate in the turbulent boundary layer the velocity profile, as shown by experiments in tubes [2, 3], can also have one of the limiting forms of Eq. (1) or Eq. (2), or the intermediate form of Eq. (3).

The velocity profile corresponding to flow of water over the plate is determined by the functions

$$\varphi = 5.85 \lg \eta + 5.56 \tag{16}$$

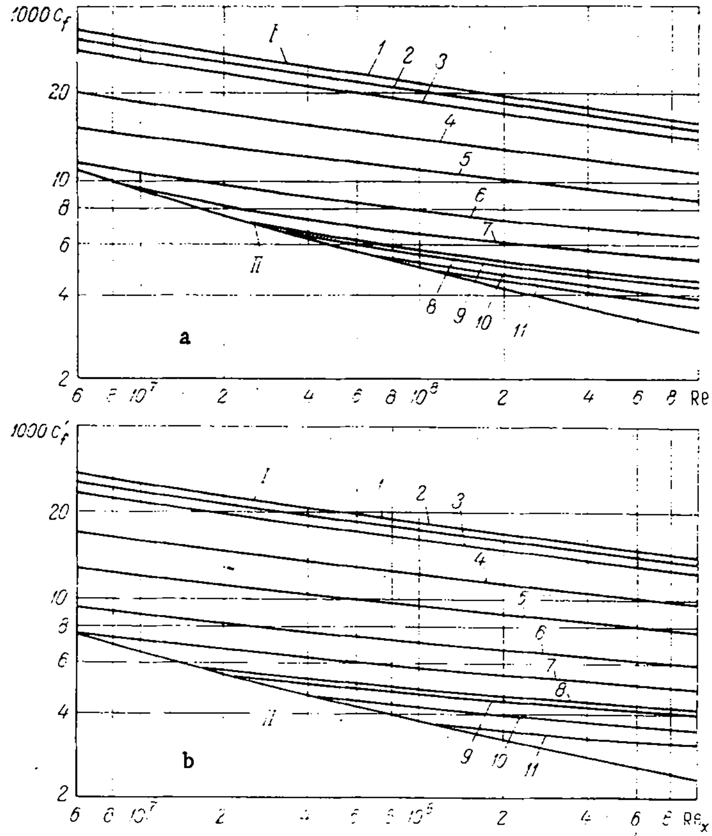

Fig. 3. Total friction coefficient of plate c_f versus $Re = U_\infty L/\nu$ (a) and c_f' versus $Re_x = U_\infty x/\nu$ (b) and parameter η_0 of the velocity profile for flow of a polymer solution on plate: 1) $\eta_0 = 15$; 2) 17; 3) 20; 4) 40; 5) 80; 6) 200; 7) 400; 8) 800; 9) 1000; 10) 2000; 11) 4000. I) Water; II) limiting case of flow over plate.

or

$$\varphi = 2.545 \ln (1 + 8.93\eta). \tag{17}$$

At sufficiently high polymer concentrations in the plate boundary layer the velocity profile saturates:

$$\varphi = 26 \ln \eta - 18.2 \tag{18}$$

or

$$\varphi = 11.2 \ln (1 + 0{,}2\eta). \tag{19}$$

For the intermediate case where the polymer concentration in the flow is less than in the case of the saturation effect, the velocity profile is also logarithmic in form, Eq. (12), but with different values of the coefficients a and b in various ranges of the coordinate η;

$$\begin{gathered} 0 < \eta \leqslant \eta_0 \quad a = 11.2, \quad b = 0.2, \\ \eta_0 \leqslant \eta < \eta_k \quad a = 2.545, \quad b = b_* - \text{ variable.} \end{gathered} \tag{20}$$

The parameter b_* is determined by the condition that Eq. (12) be satisfied by condition (20) at the breakpoint of the velocity profile at coordinate η_0;

$$5.85 = 2.305a, \quad 20.25 \lg \eta_0 - 18.2 = a \ln b_*. \tag{21}$$

To determine the dependence of the fluid friction coefficients c_f and c_f' on Reynolds number Re for the limiting cases of flow in the boundary layer, where the velocity profiles correspond to a water flow ($a = 2.545$, $b = 8.93$), or to the limiting case of friction reduction in the boundary layer ($a = 11.2$, $b = 0.2$), Eq. (15) is used in unchanged form with the corresponding values of the coefficients a and b. In case (20) the functions $\Psi(z)$ and $\Phi(z)$ are determined by the same Eq. (15), but with consideration of the change in coefficients a and b in the different sections of the logarithmization.

The equations presented above, Eqs. (5)-(20), allow graphic presentation (Fig. 3a,b) of the dependence of complete and local friction coefficients on Reynolds number and the parameter η_0 of the velocity profile on the back edge of the plate (when calculating the total friction coefficient) or at the current x coordinate (when calculating the local friction coefficient). The frictional-drag data are presented in more convenient form in Fig. 2, as a function of the degree of change in velocity profile ($\eta_0 - 15$). It is evident from the figure that with increase in Re the character of this dependence does not change, the entire curve shifting somewhat in the direction of larger Reynolds numbers and reaching the large value of limiting resistance reduction, which is a consequence of the assumption of the existence of a limiting form for the velocity profile.

In cases of practical importance the conditions for flow over a plane plate are such that the shear stress values on the wall $\tau > 300$ dyn/cm², which corresponds to the saturation regime of the drag-reduction effect at the chosen polymer concentration value. Under these conditions the profile parameter η_0 = const is realized with flow over the plate of a homogeneous polymer solution of concentration $c = k(\eta_0 - 15)$ = const, where the polymer effectiveness coefficient k is determined by results of experiments in tubes.

For a single polymer the data of Fig. 2 permit comparison of the effectiveness of homogeneous polymer solutions of differing concentrations for flow in a tube or flow over a plate. the character of this dependence for the two types of flow is identical, but in flow on a plate the reduction in total fluid friction is 10-15% less than for flow in a tube. With increase in Re the entire curve shifts somewhat toward higher Reynolds numbers and ensures a high value of maximum drag reduction.

The same data of Fig. 2 for different polymers permit quantitative comparison of the effectiveness of polymer solutions of various concentrations.

The proposed method for calculating frictional drag of a plate in a homogeneous polymer solution in contrast to other known methods for this calculation [9,10], produces a simple graphic dependence between effectiveness and the concentration of a polymer of any type, for which the effectiveness coefficient k is known.

NOTATION

L, plate length; B, width; δ, boundary layer thickness; y, distance from wall; x, distance from forward edge of plate; U_∞, velocity of incident flow at infinity; u, flow velocity in boundary layer at distance y from wall; $v_* = \sqrt{\tau_0/\rho}$, dynamic velocity; τ_0, shear stress on wall; ρ, density of water; ν, kinematic viscosity; $\eta = yv_*/\nu$, dimensionless value of distance from plate wall; η_0, velocity profile parameter with polymer present in flow; $\eta_k = \delta v_*/\nu$ at x = L, boundary layer thickness at back edge of plate in dimensionless form; $\varphi = u/v_*$, dimensionless velocity in plate boundary layer at distance y; c, weight concentration of polymer in boundary layer; λ, fluid friction coefficient in tube; c_f, total friction coefficient on plate; c_f', local friction coefficient on plate; $S = (\lambda_w - \lambda_p)/\lambda_w$, $S = (c_{f,w} - c_{f,p})/c_{f,w}$ at Re = const, drag-reduction for flow in tube or flow over plate, respectively. Indices w and p indicate water and polymer solutions, N indicates numerically determined variables.

LITERATURE CITED

1. Yu. F. Ivanyuta and L. A. Chekalova, Inzh.-Fiz. Zh., 24, No. 5 (1974).
2. P. S. Virk, J. Fluid Mech., 45, Part 3 (1971).
3. Yu. F. Ivanyuta and L. A. Chekalova, Inzh.-Fiz. Zh., 35, No. 2 (1976).
4. Yu. F. Ivanyuta and L. A. Chekalova, Inzh.-Fiz. Zh., 21, No. 1 (1971).
5. P. S. Virk, E. W. Merrill, H. S. Micley, and K. A. Smith, J. Fluid Mech., 30, Part 2 (1967).
6. A. S. White, J. Mech. Eng. Sci., 8, No. 4 (1966).
7. J. W. Hoyt, Trans. ASME, June, 258 (1972).
8. H. Schlichting, Boundary Layer Theory, McGraw-Hill (1968).
9. N. G. Vasetskaya and V. A. Ioselevich, Sb.Nauchn. Tr. Inst. Mekh. Mosk. Gos. Univ., No. 32 (1974).
10. V. A. Gorodtsov, Inzh.-Fiz. Zh., 31, No. 3 (1976).